Theorem. Let  be an invertible matrix,  the cofactor of element , and adj *A* =  the adjugate matrix of *A*. Then .

Proof.

The **adjugate**  is defined as  where the minor  is the determinant of the matrix obtained from A by removing row *i* and column *j*.

The (*i*, *j*) entry of *A* adj *A*

= 

=  (1)

The (*i*, *i*) entry of *A* adj *A =*  = 

since this is the cofactor expansion of *A* along row *i*.

We will show that the (*i*, *j*) entry of *A* adj *A* = 0 if *i* ≠ *j*.

Start by generating a matrix *B* from *A* by replacing row *j* by row *i*:



The cofactor of the (*j*, *k*) element of matrix *B* is  times the determinant of the matrix highlighted in magenta.

Next re-write matrix *A* so as to compare its (*j*, *k*) cofactor to that of *B*.



Observe that the magenta cofactor is the same. That is,  is the common cofactor for matrices *A* and *B* as long as attention is restricted to row *j*.

Now generate the determinant of *B* by expanding *B* by cofactors along row *j*. Since *B* has two identical rows, its determinant is zero. From equation (1) we see that



 (*i*, *j*) entry of *A* adj *A*

Therefore



Left multiplying both sides by  and dividing both sides by  yields

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